TECHNICAL NOTE

PITFALLS OF POLYNOMIALS

SECOND ORDER RESISTOR VOLTAGE COEFFICIENTS THE PITFALLS

A number of chip processes model resistors using second order polynomials. This approach can lead to a situation where a circuit can have multiple solutions only one of which is correct. This note explains why and what you can do to avoid this problem.

Essentially the i/v relationship of a resistor with quadratic terms is:

$$i = \frac{v}{r_0 \times (1 + c_1 \cdot abs(v) + c_2 \cdot v^2)}$$

where:

c_1 is the first order coefficient

c_2 is the second order coefficient

v is the resistor's terminal voltage

r_0 is the value of the resistor when $v = 0$

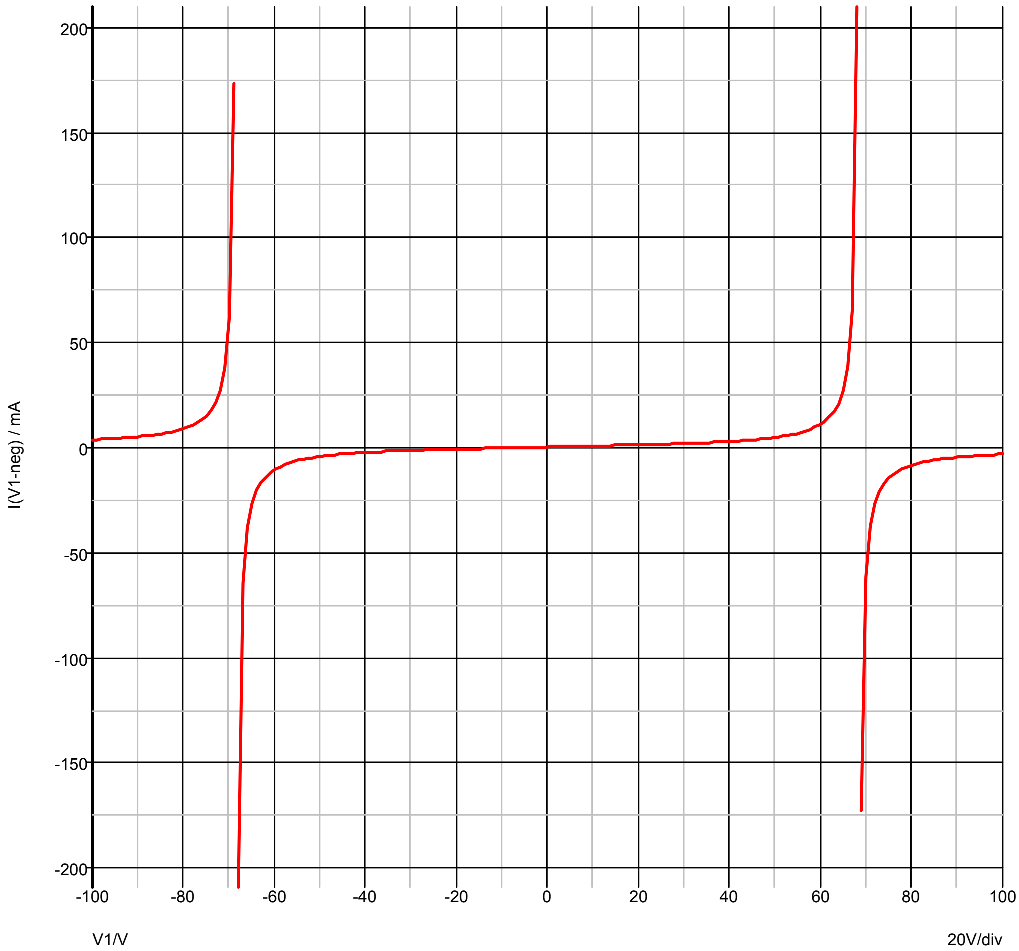
Let's consider a resistor with the following parameters:

$c_1 = 8.03e^{-3}$

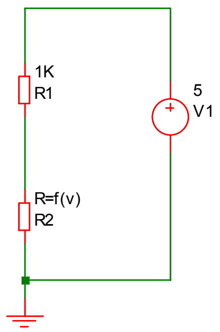
$c_2 = -3.32e^{-4}$

$r_0 = 19240$

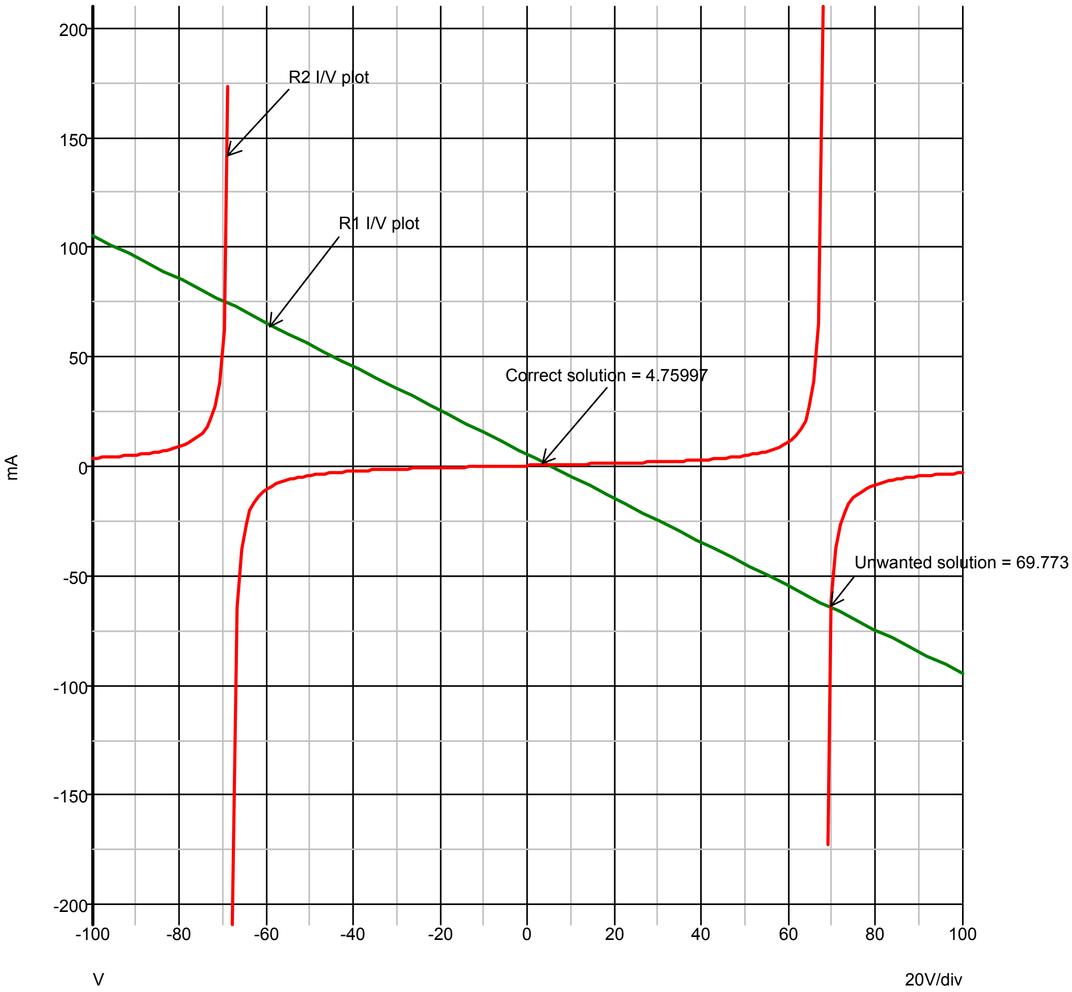
The following graph show the relationship between resistance and applied voltage:



As can be seen, at about 70V the current reverses. This is a problem because it means that a circuit using such a device can have more than one mathematically correct solution. Consider this simple circuit:



R1 is a normal linear resistor and R2 is the non-linear device as described above. The following graph shows a graphical method of finding the solution to the circuit:



The green line represents the current-voltage characteristic of the combination of V1 and R1 while the red curve represents the current-voltage characteristic of the non-linear resistor R2. The solution to the circuit is where the curves intersect. As can be seen this happens in three different places meaning that there are three possible solutions.

The expected and ‘correct’ solution is at V=4.75997. As shown, there is also a solution at V=69.773.

Exactly what solution the simulator will find is not always easy to control. Usually it *will* find the expected solution but this can’t always be guaranteed.

This situation arises in the above example because c2 is negative and there therefore exists a real value of v for which the resistance falls to zero. (In fact there remains a problem even if c2 is positive, as although the current won't change sign, its slope still does change sign and therefore multiple solutions at a fixed current remain)

The solution to this problem is to define the resistor to be better behaved. Here is an alternative:

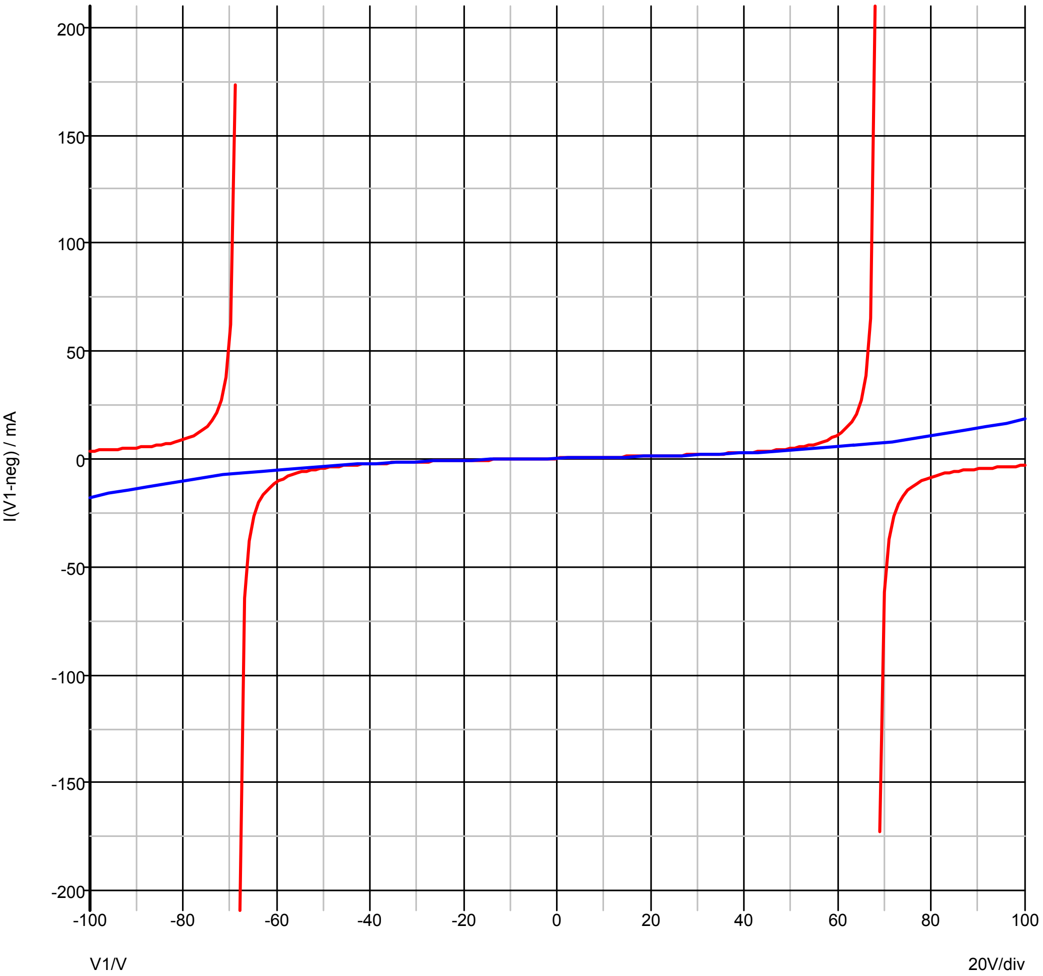
$$i = \frac{v \times (1 - c_1 \times abs(v) - c_2 \times v^2)}{r_0}$$

For small values of voltage this is in fact a close approximation to the original equation as in general:

$$\frac{1}{1+\delta} \approx 1 - \delta$$

provided that $\delta \ll 1$

The following curve show a plot of the modified equation alongside the original:



The blue curve is the new modified equation while the red curve show the original version.

The above formulation can still suffer difficulties depending on the values of c1 and c2. If the following inequality is true then the device will be well behaved:

$$c_1^2 + 4 \times c_2 < 0$$